Worksheet on Chapters 1 and 2

1 Functions

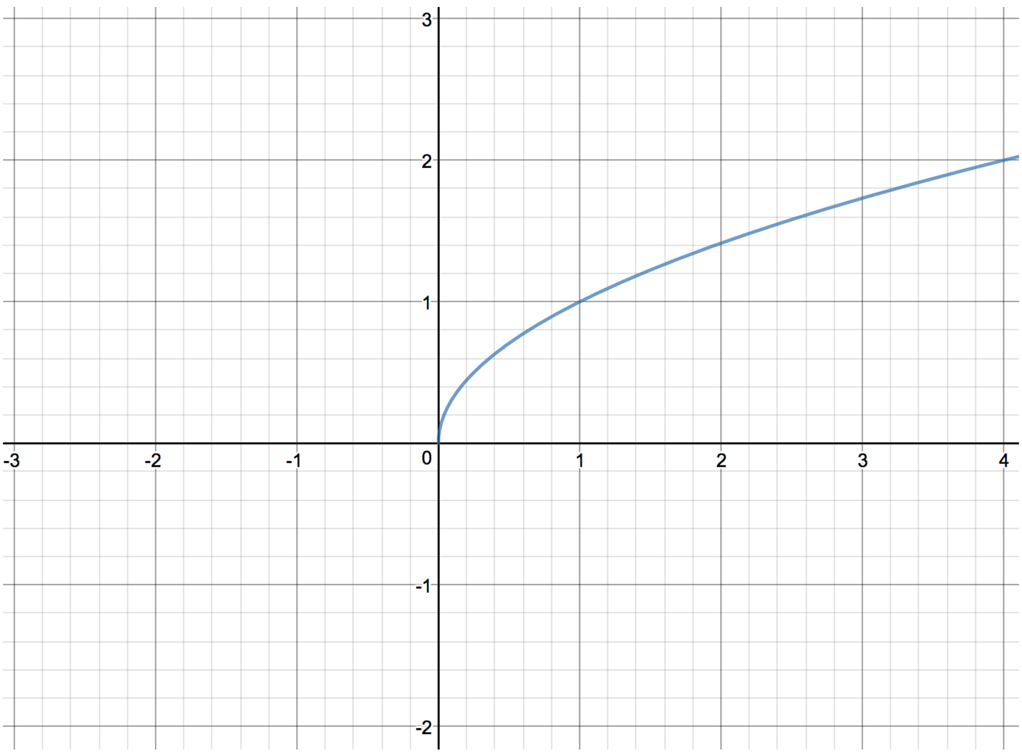
Problem 1. Consider the function $f(x) = 5x + 7$. Which of the following is true

1. The graph of $f(x)$ does not intersect the line the $y = x$ and therefore it does not have an inverse function
2. The inverse function $f^{-1}(x)$ exists and therefore its graph never touches the graph of $f(x)$
3. The graph of $f^{-1}(x)$, the graph of $f(x)$ and the line $y = x$ all meet in exactly one point
4. The function $f^{-1}(x)$ exists, but only after restricting the domain of $f(x)$ appropriately

Problem 2. Find the implied domains of the following functions

1. $f(x) = \sqrt{73 - x} - \sqrt{37 + x}$
2. $f(x) = 5\ln(x - 6)$

Problem 3. Sketch the graph of $y = -\sqrt{2x - 1} + 1$ starting from the graph of $y = \sqrt{x}$.



2 Limits

Problem 4. Multiple Choice. Consider the function

$$f(x) = \begin{cases} x^2 & x \text{ rational} \\ -x^2 & x \text{ irrational} \\ \text{undefined} & x = 0 \end{cases}$$

Then

1. There is no a for which $\lim_{x\to a} f(x)$ exists
2. There may be an a for which $\lim_{x\to a} f(x)$ exists, but we can't say what it is without more information
3. $\lim_{x\to a} f(x)$ exists for $a = 0$
4. $\lim_{x\to a} f(x)$ exists for infinitely many a

Problem 5. True or False. The limit $\lim_{x\to a} f(x)$ depends on how $f(a)$ is defined.
Problem 6. True or False. If $f(a)$ is undefined then $\lim_{x\to a}$ cannot exist.
Problem 7. If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} f(x)/g(x)$

1. Does not exist
2. Must exist
3. Not enough information

Problem 8. $\lim_{x\to 0} x^2 \sin(1/x)$

1. Does not exist because no matter how close x gets to 0, there are x's near zero for which $\sin(1/x)$ is 1, and x's for which $\sin(1/x)$ is -1
2. Does not exist because the function value oscillates around 0
3. Does not exist because $1/0$ is undefined
4. Equals 0
5. Equals 1

Problem 9. Find the all the asymptotes of the function $f(x) = \dfrac{1+x^4}{x^2-x^4}$.

3 Continuity

Problem 10. You are running a bath but you don't close the tap properly and it is dripping. It drips once per second, each drip raising the level of the bathwater by exactly 1mm.

1. Let f be the function that represents hight of the bathwater at time t. Is $f(x)$ a continuous function?

2. Let g be the function that describes the volume of water as a function of the height of the bathwater. Is $g(x)$ a continuous function?

Problem 11. You know that

If $f(x)$ is a polynomial function then $f(x)$ is continuous.

Which of the following is true.

1. If $f(x)$ is continuous then $f(x)$ is a polynomial

2. If $f(x)$ is not a polynomial then $f(x)$ is not continuous

3. If $f(x)$ is not continuous then $f(x)$ is not a polynomial

4. All of the above

Problem 12.

1. Solve the equation $x^2 + 13x + 41 = 1$.

2. Use the IVT to prove that $x^2 + 13x + 41 = \sin x$ has at least 2 solutions between the two roots found above.